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(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, JANUARY 2015

FIRST YEAR

MATH FOR ECO (General)

Paper : I

Date : 14/01/2015

Time : 11 am – 2 pm

Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

(***Answer any seven questions***) [7×5]

1. $A=\{x\in\mathbb{R} \mid x\in[2,5]\}$ & $B=\{x\in\mathbb{R} \mid x\in[4,5]\}$ Then $(A^C-B)^C=?$ [5]
2. Check whether the following function is injective and surjective? $f(x)=x^2+2,\ x\in\mathbb{R}$. [3+2]
3. Prove that the finite intersection of open sets in $\mathbb{R}$ is also open in $\mathbb{R}$. [5]
4. Prove that a monotone increasing sequence which is bounded above, converges to its supremum. [5]
5. Prove that every convergent sequence is a cauchy sequence. [5]
6. Prove that the series $1+\frac{1}{2}+\frac{1}{3}+...$ is not convergent. [5]
7. Test the convergence of the series $\sum\frac{n^2}{2^n}$. [5]
8. Prove that an absolutely convergent series is convergent. [5]
9. Prove that every cauchy sequence in $\mathbb{R}$ is bounded. [5]
10. Prove that the set A – B is an open set, where $A=(0,1)$ and $B=\left\{\frac{1}{2^n};n\in\mathbb{N}\right\}$. [5]

Group – B

(***Answer any eight questions***) [8×5]

11. Reduce the following matrix to a roco-reduced echelon form and then find its rank : [5]

$$\begin{bmatrix} -1 & 2 & -1 & 0 \\ 2 & 4 & 4 & 2 \\ 0 & 0 & 1 & 5 \\ 1 & 6 & 3 & 2 \end{bmatrix}$$

12. For what values of a & b the following system of equations has (i) a unique solution, (ii) infinite solutions, (iii) no solution.
    $x_1+4x_2+2x_3=1,\ 2x_1+7x_2+5x_3=2b,\ 4x_1+ax_2+10x_3=2b+1$. [5]
13. Show that A_3 is a normal subgroup of S_3 [5]
14. Prove that the set $\langle Z_5,\oplus_5,\otimes_5\rangle$ forms a commutative ring with unity. Is it a field? [5]
15. Using De Moivre's theorem prove that if α,β,γ are reals and $\cos\alpha+\cos\beta+\cos\gamma=0$ & $\sin\alpha+\sin\beta+\sin\gamma=0$ then, [5]
    a) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$
    b) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$
16. Using De Moivre's theorem solve the equation: $x^7+1=0$. [5]
17. Show that the set of all even integers forms a commutative ring w.r.t usual addition & multiplication. [5]

18. a) Show that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is an orthogonal matrix. [3]

b) Show that determinant of any orthogonal matrix is either +1 or –1. [2]

19. Solve the difference equation : $u_{n+3} + u_{n+2} - 8u_{n+1} - 12u_n = 0$ [5]

20. Find u_x, from the following equation : $u_{x+2} + u_x = \sin\frac{\pi}{2}x$ [5]

21. Show that $\Delta = E - 1$ where, $\Delta f(x) = [f(x+h) - f(x)]$ and $Ef(x) = f(x+h)$. [5]

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